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Classical and Quantum Heuristics for the Binary Paint Shop Problem

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Read the paper

arxiv.org/abs/2509.15294

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The Binary Paint Shop Problem

Each car model appears twice in a fixed sequence: paint one copy red and the other blue. Finding the colouring with the fewest paint swaps is APX-hard.

We want to develop heuristics that minimise

$$\mathbb{E}\left(\frac{\Delta_C}{n}\right) = \mathbb{E}\left[\frac{\#\text{ of paint swaps}}{\#\text{ of car models}}\right]$$

Why it matters

Fewer swaps reduce overhead costs and improve efficiency and profit in paint shops.

Classical Heuristics for the BPSP

Red-First

First copy red,
second copy blue.

7 swaps

Greedy

Keep the colour until
forced to switch.

6 swaps

Recursive Greedy

Remove pairs, then reinsert
with the fewest added swaps.

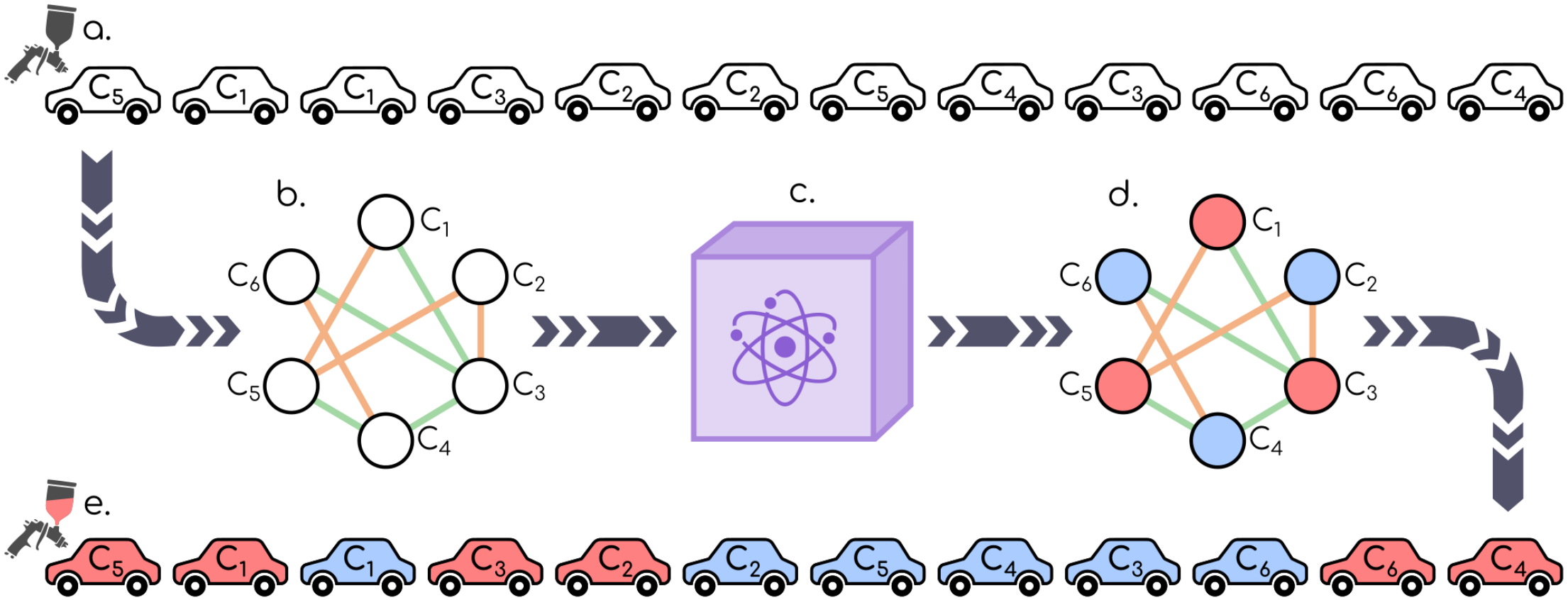
5 swaps

Recursive Star Greedy

Leave tied colour choices
undecided until later.

4 swaps

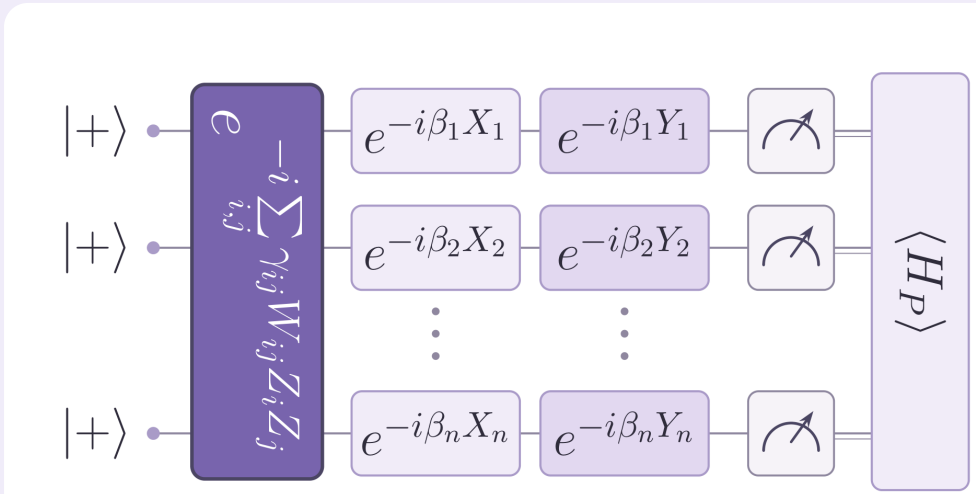
Solving the BPSP on a Quantum Computer



Each model's two cars must have opposite colours, so one binary variable determines both. We map these variables to a weighted graph-partitioning problem, then use quantum optimisation to minimise colour swaps.

Expressive QAOA (XQAOA)

XQAOA gives shallow quantum circuits more flexibility through independently tunable angles.



V. Vijendran, A. Das, D. E. Koh, S. M. Assad, and P. K. Lam,

An Expressive Ansatz for Low-Depth Quantum Approximate Optimisation,

Quantum Sci. Technol. 9, 025010 (2024).

Cost and Mixing Hamiltonians

$$H_P = \sum_{(i,j)\in E} J_{ij} Z_i Z_j$$

Encodes the objective function.

$$H_M^X = \sum_j X_j \qquad H_M^Y = \sum_j Y_j$$

Mixes candidate solutions.

Prepared State

$$|\psi(\boldsymbol{\gamma},\boldsymbol{\beta})\rangle = \prod_{j=1}^{n} e^{-i\beta_j Y_j} e^{-i\beta_j X_j} \prod_{(i,j)\in E} e^{-i\gamma_{ij} J_{ij} Z_i Z_j} |+\rangle^{\otimes n}.$$

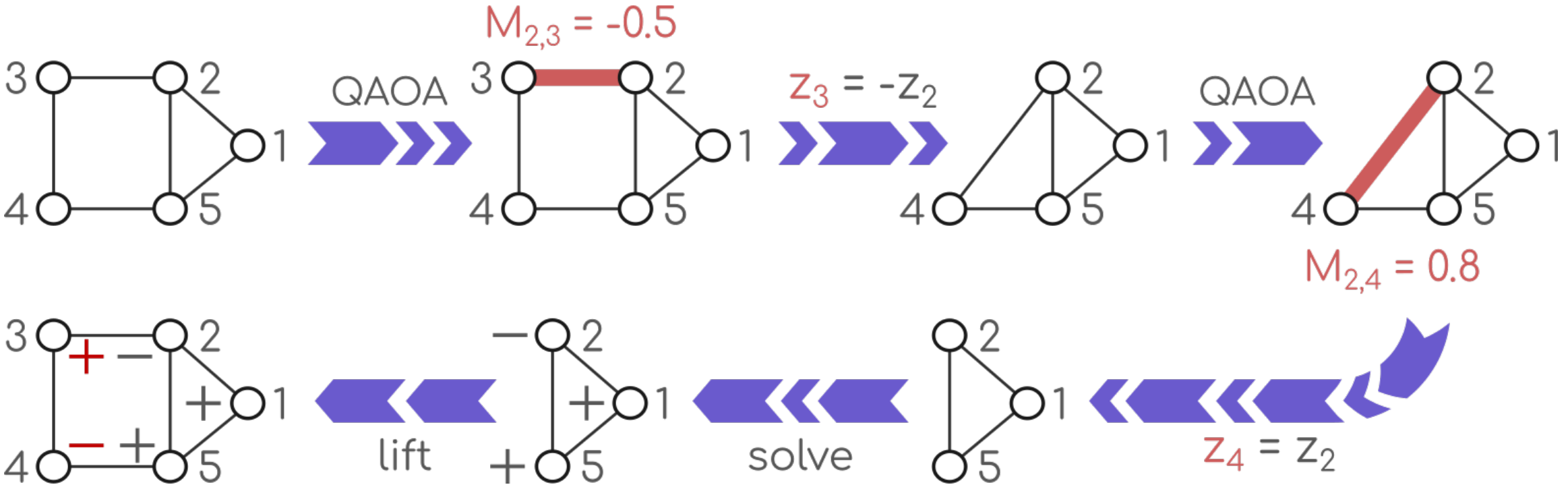
Rotations convert cost phases into outcome probabilities.

Classical Optimisation

Tune the angles to minimise the expected paint-swap ratio.

$$\min_{\boldsymbol{\gamma},\boldsymbol{\beta}} \langle\psi| H_P |\psi\rangle$$

Recursive QAOA (RQAOA)

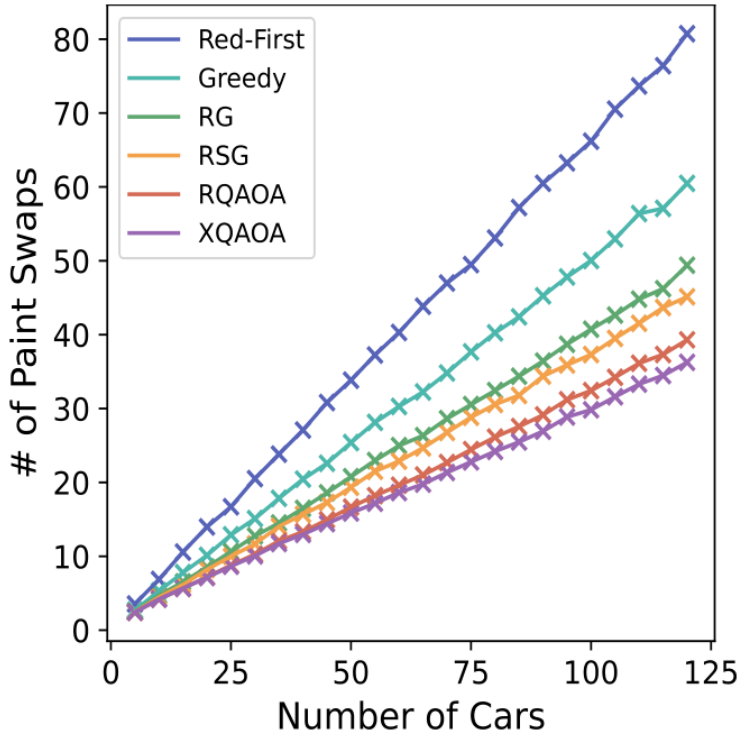


RQAOA uses QAOA as a subroutine to recursively reduce the problem until it is small enough to solve exactly.

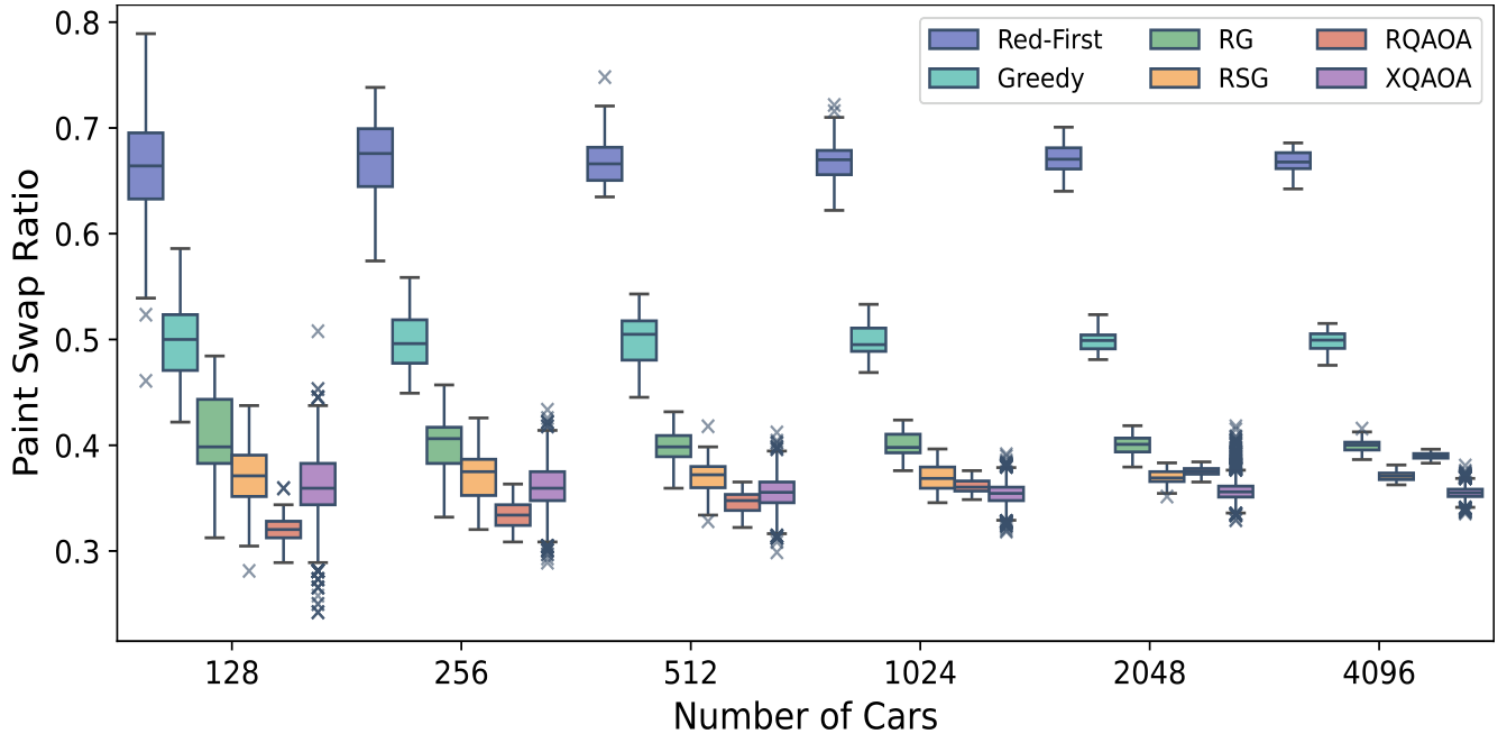
S. Bravyi, A. Kliesch, R. Koenig, and E. Tang,
Obstacles to State Preparation and Variational Optimization from Symmetry Protection,
Phys. Rev. Lett. 125, 260505 (2020).

Benchmarking up to 4,096 Qubits

Classical simulations using analytic expectations: 200 random instances per size (5–120 models), 50 per size (128–4,096). XQAOA: best of 100 restarts (left), all 100 runs (right).



RSG leads the classical methods.
XQAOA has the lowest mean count.

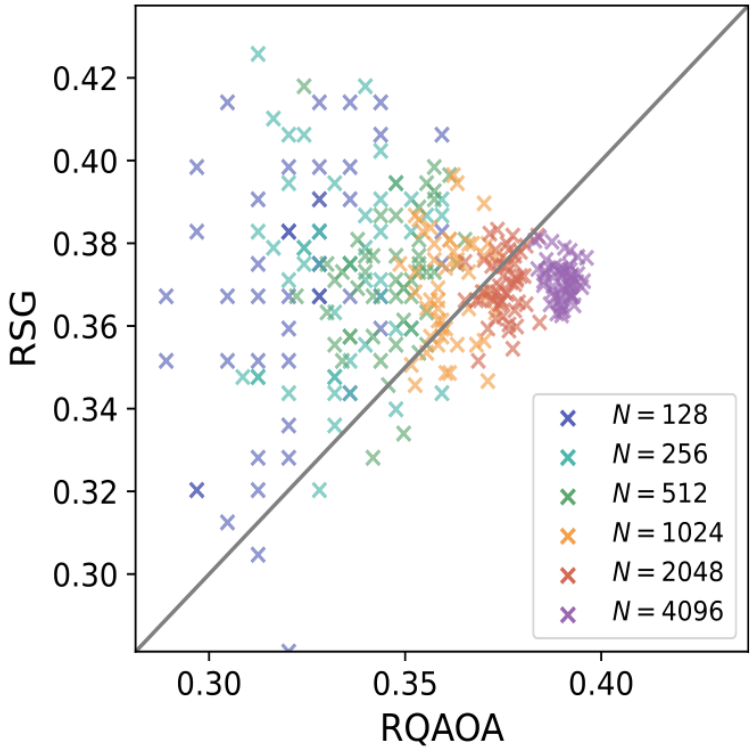


RSG remains the strongest classical baseline.
XQAOA stays near 0.357 as RQAOA deteriorates.

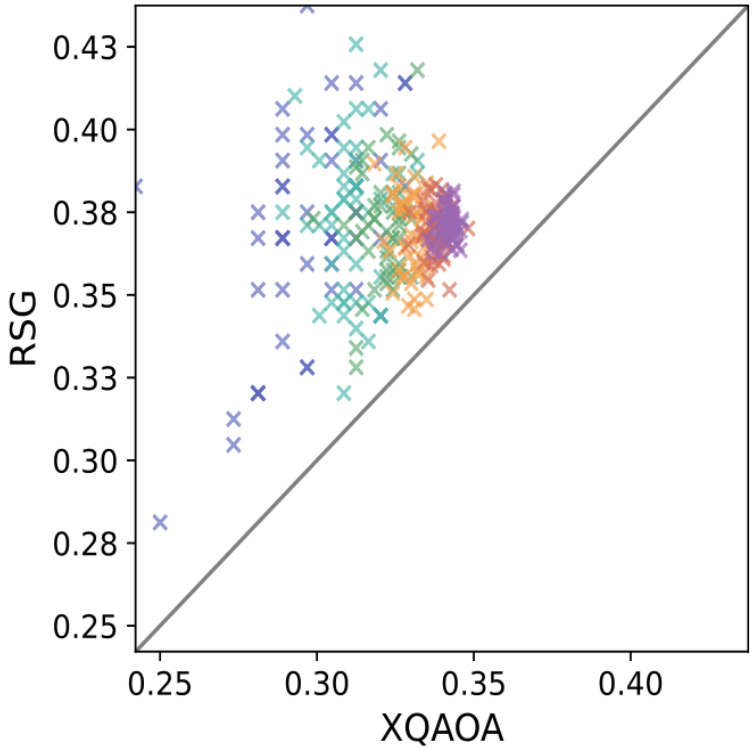
RQAOA falls behind RSG from 2,048 qubits. XQAOA stays ahead of RSG through 4,096.

Instance-by-Instance Comparisons

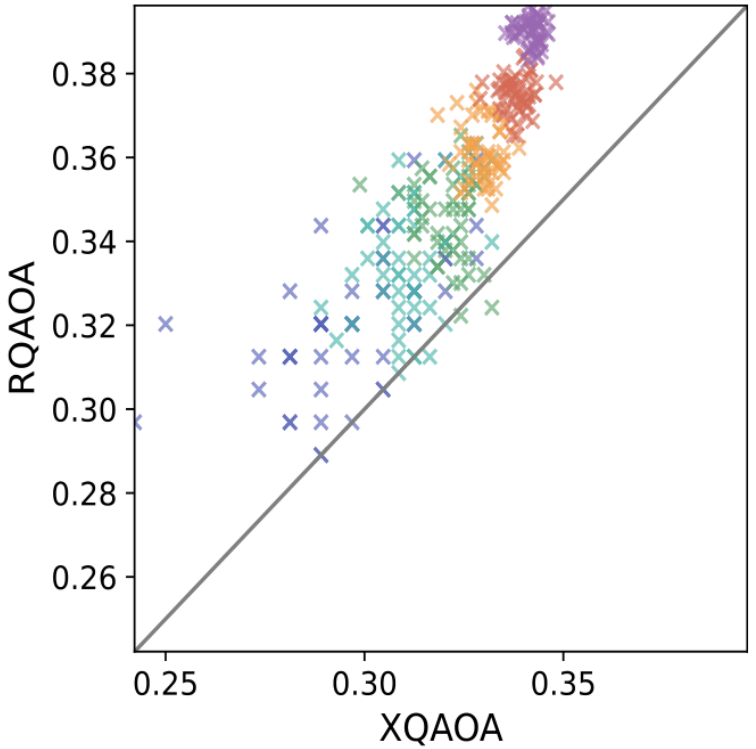
On the diagonal, both methods tie. Above it, the horizontal-axis method uses fewer swaps.



RQAOA's advantage over RSG reverses as size grows.



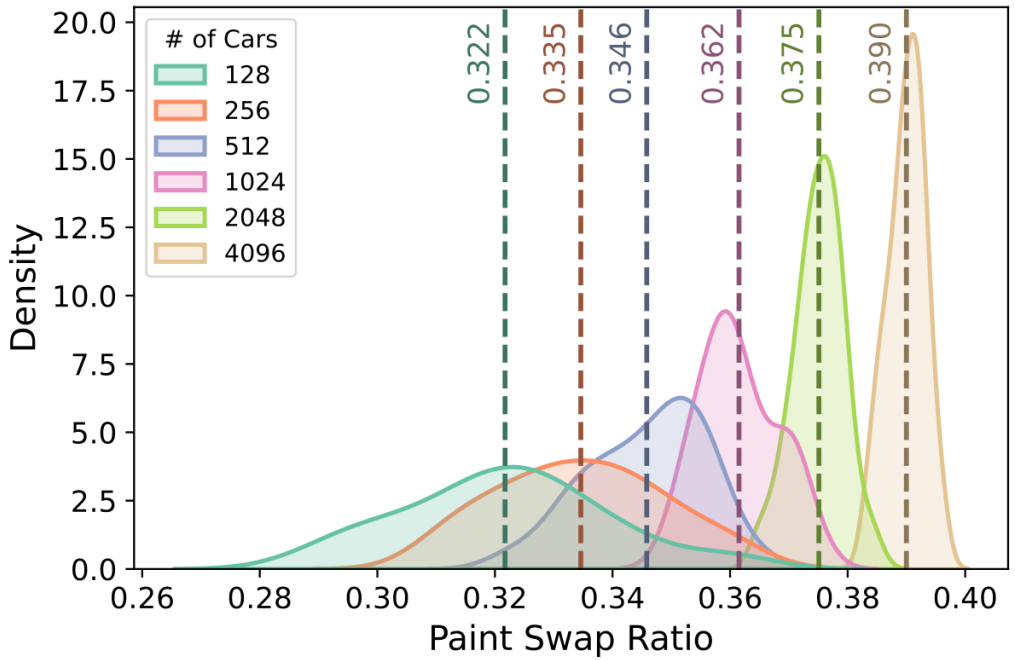
XQAOA beats RSG on every tested instance.



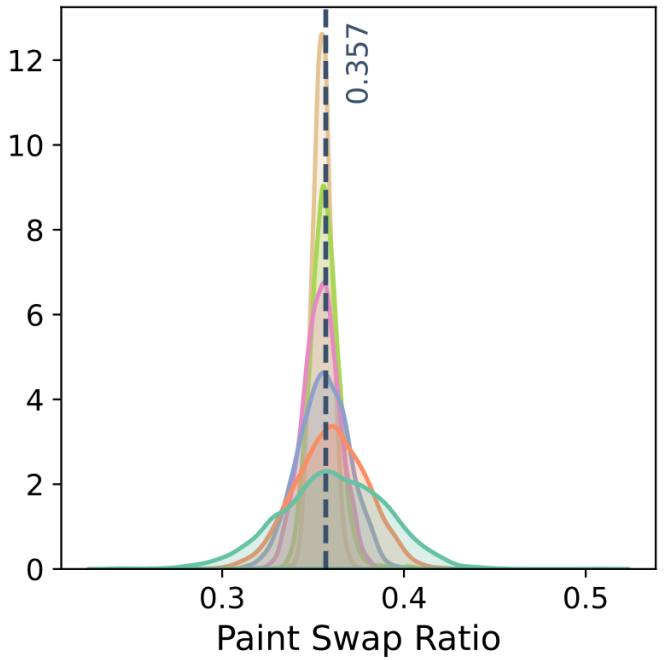
XQAOA beats RQAOA on almost every instance.

Diminishing Returns of Level-1 Recursive QAOA

RQAOA Mean 0.322 → 0.390



XQAOA Stable at 0.357



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Read the paper: arxiv.org/abs/2509.15294

RQAOA deteriorates with size, despite **provably near-optimal** QAOA angles at every step!