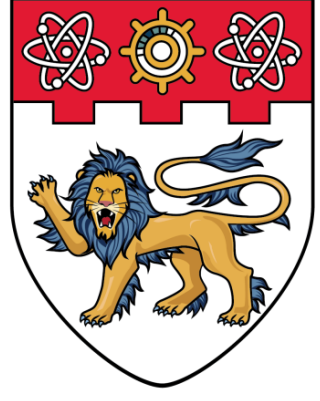




Topological Structures in Multiphoton Entanglement

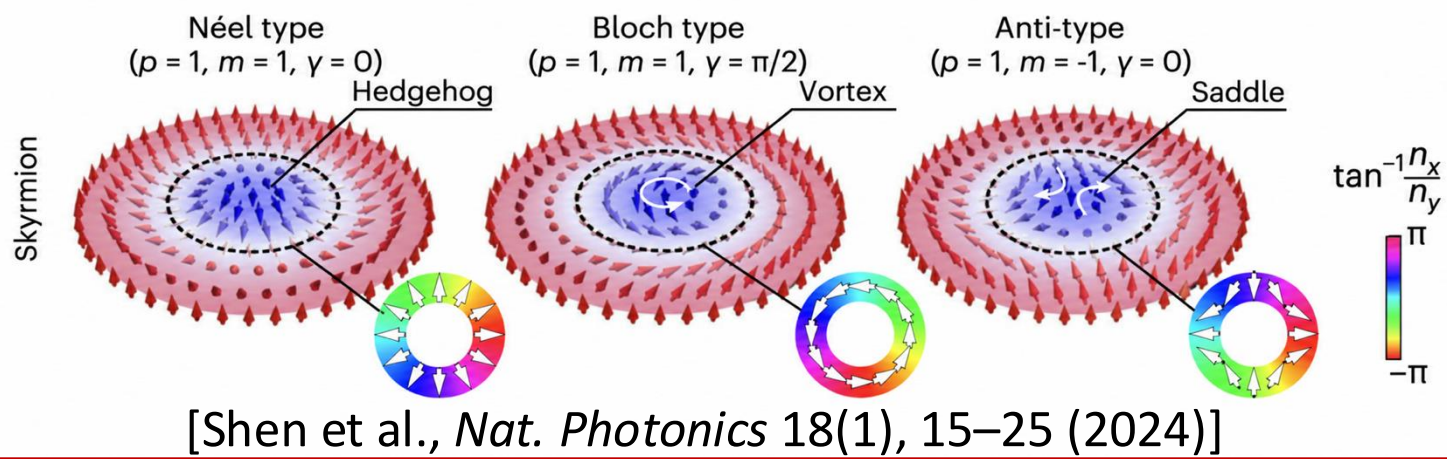
Introduction

Skyrmion

- **Topologically stable** vector texture
- Characterized by a **topological invariant**

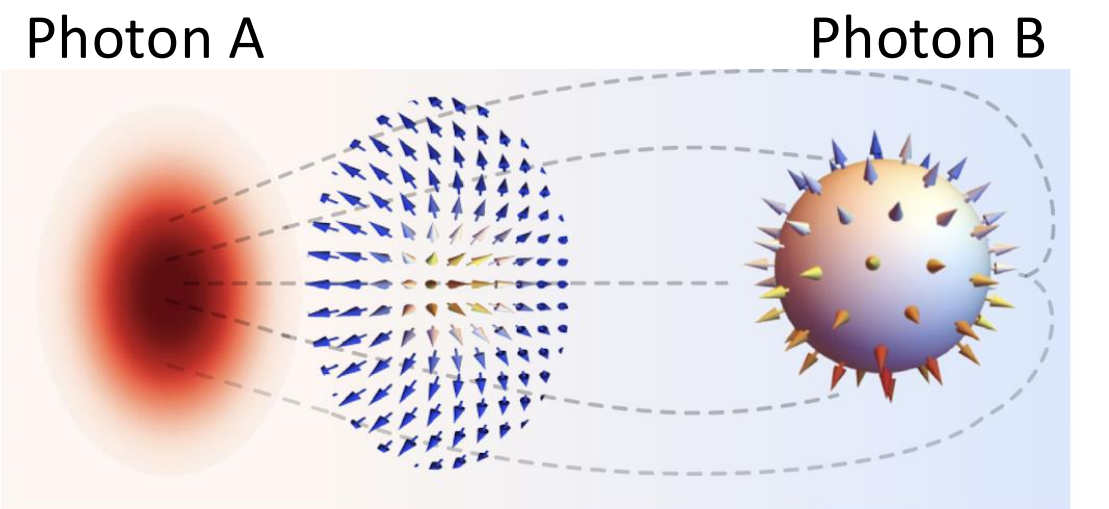
Skyrmion number

$$N = \int \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial x} \times \frac{\partial \vec{S}}{\partial y} \right) dx dy$$



Hybrid entanglement

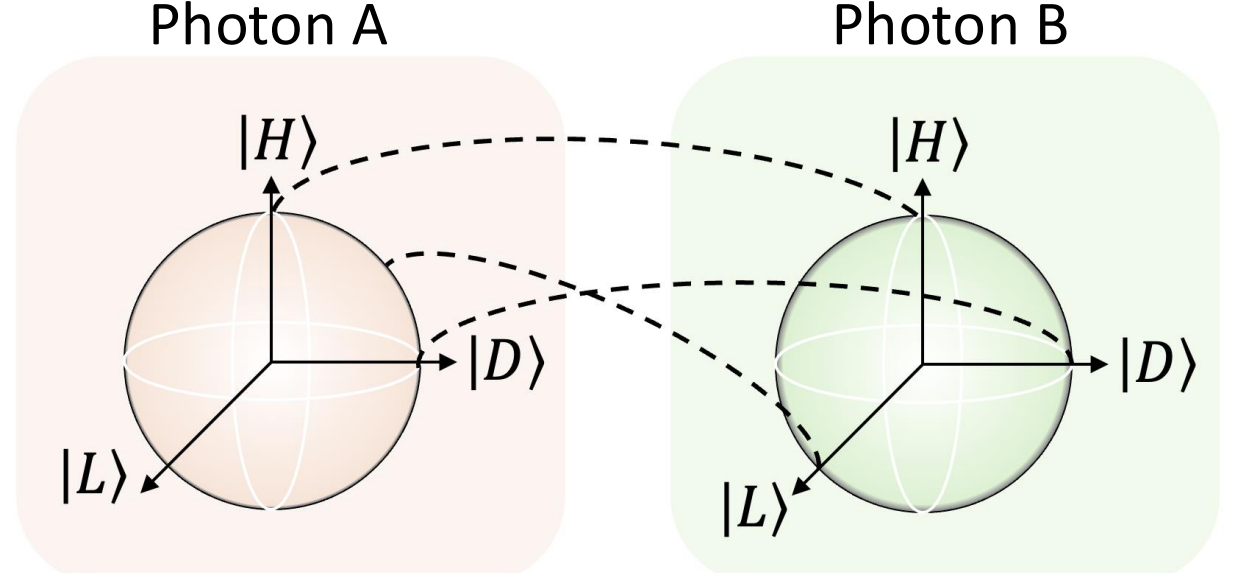
OAM-Polarization entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\ell_1 H\rangle + |\ell_2 V\rangle)$$

Polarization entanglement

Polarization-Polarization entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|HV\rangle + |VH\rangle)$$

Beyond two photons

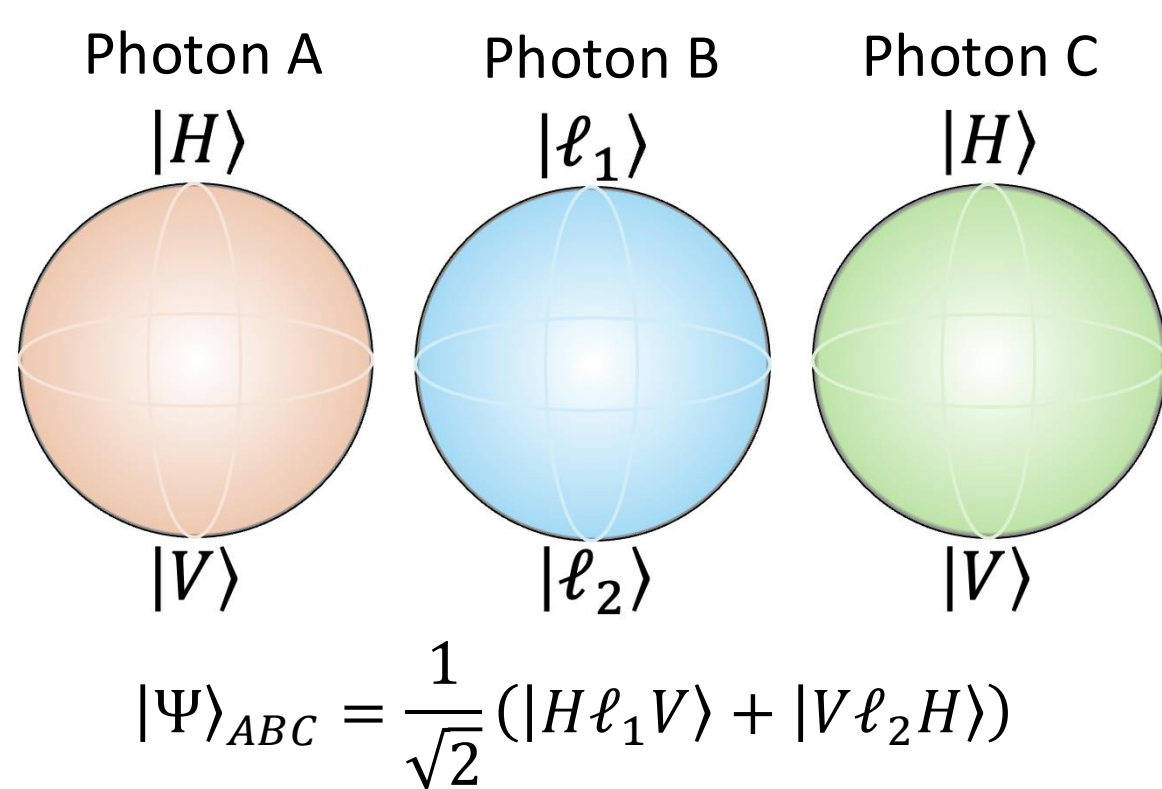
Extending two-photon topology to three-photon entanglement

We propose a framework to reveal topological structures in three-photon entangled states

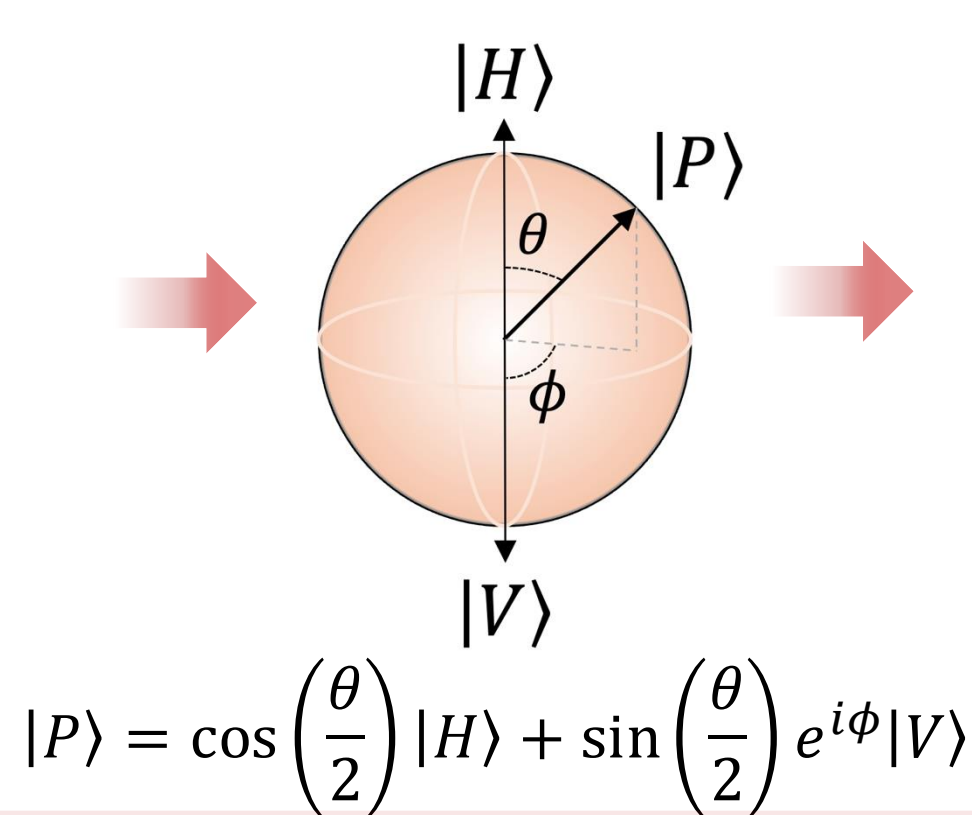
Approach

Projecting photon A onto $|P\rangle$ prepares photon B and C in a **conditional entangled state** from which a quantum skyrmion texture can be extracted

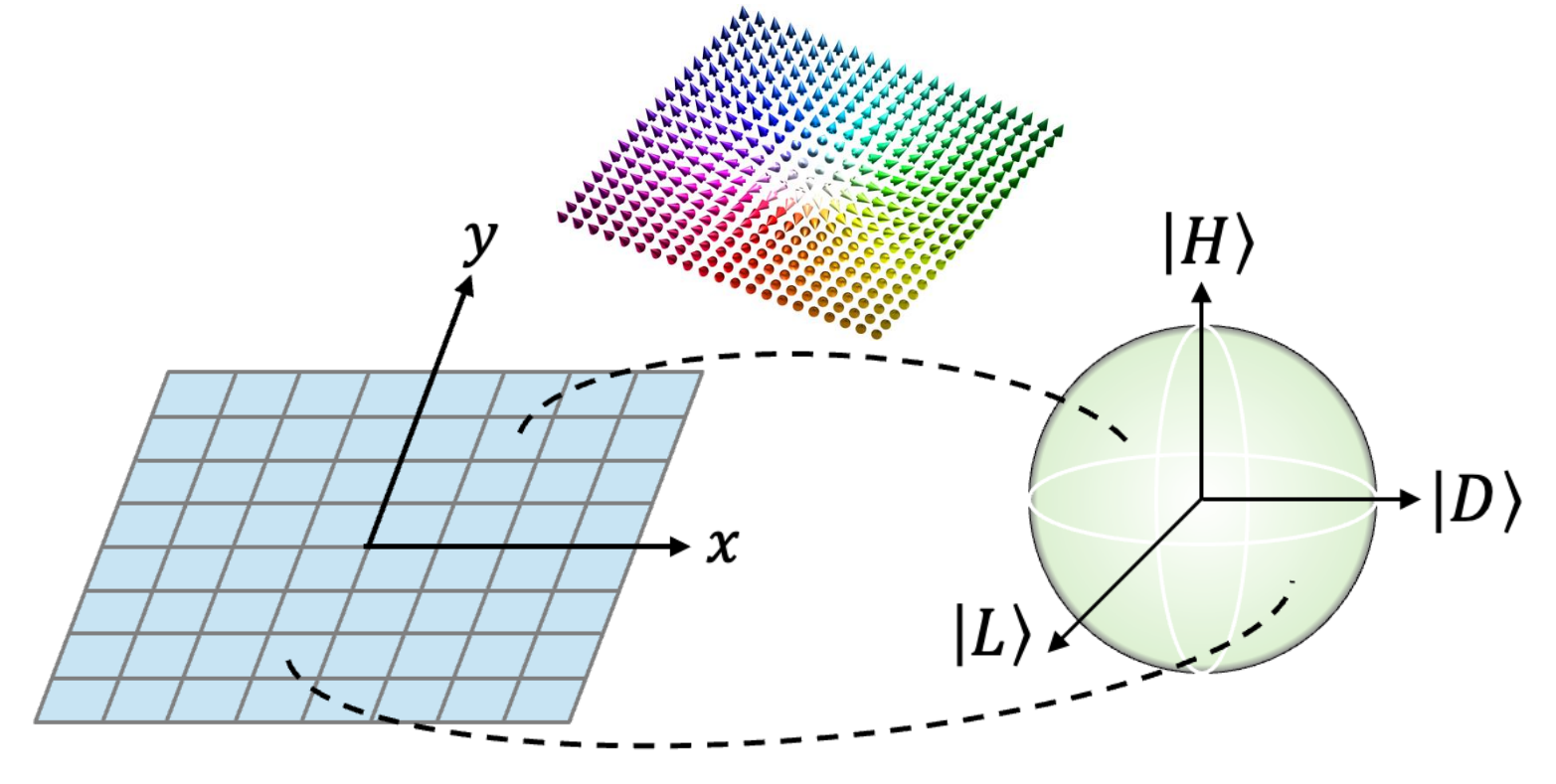
Three-photon entanglement



Project photon A



Conditional quantum skyrmion



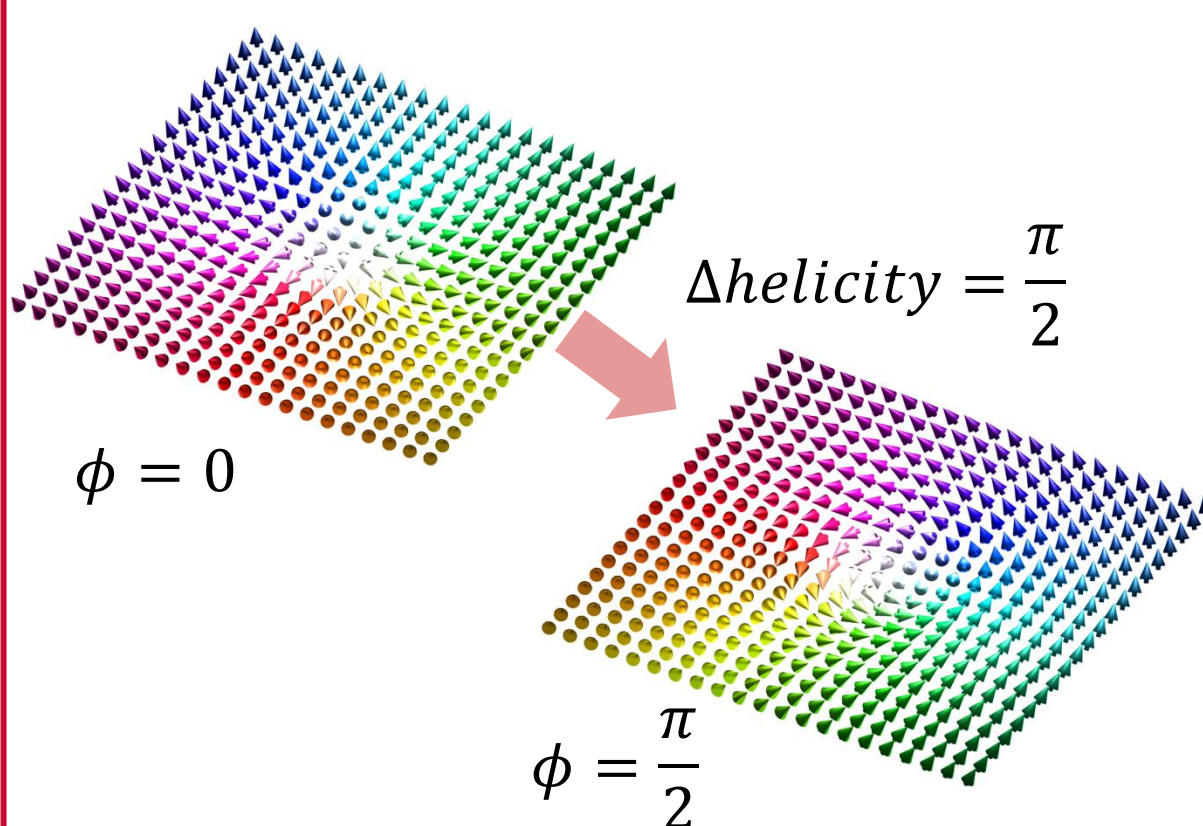
Conditional state

$$|\psi\rangle_{BC} = \cos\left(\frac{\theta}{2}\right) |\ell_1 V\rangle + \sin\left(\frac{\theta}{2}\right) e^{i\phi} |\ell_2 H\rangle$$

Revealing Hopfion topology

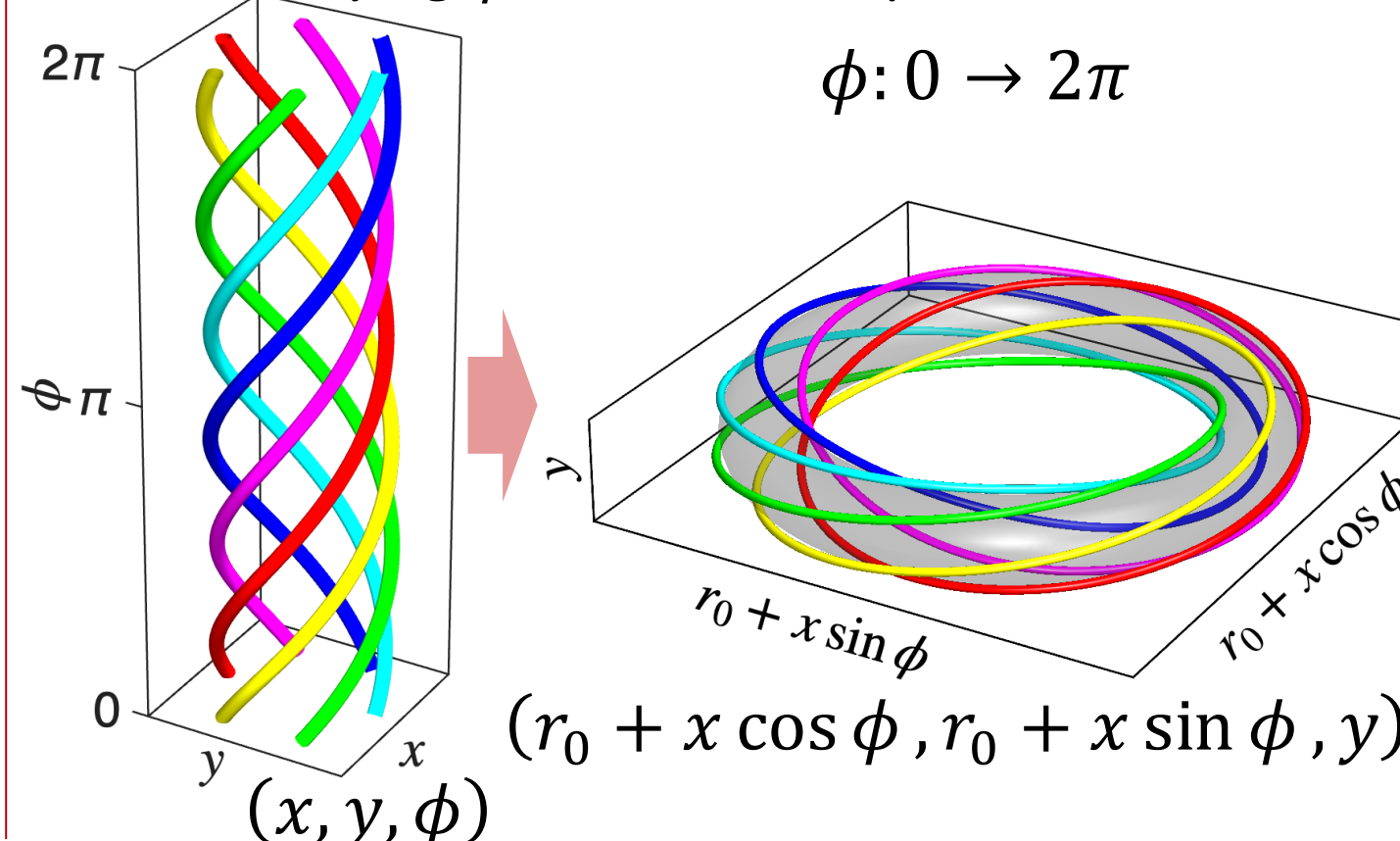
Phase-controlled helicity

The helicity of the skyrmion can be controlled by the phase of $|P\rangle$

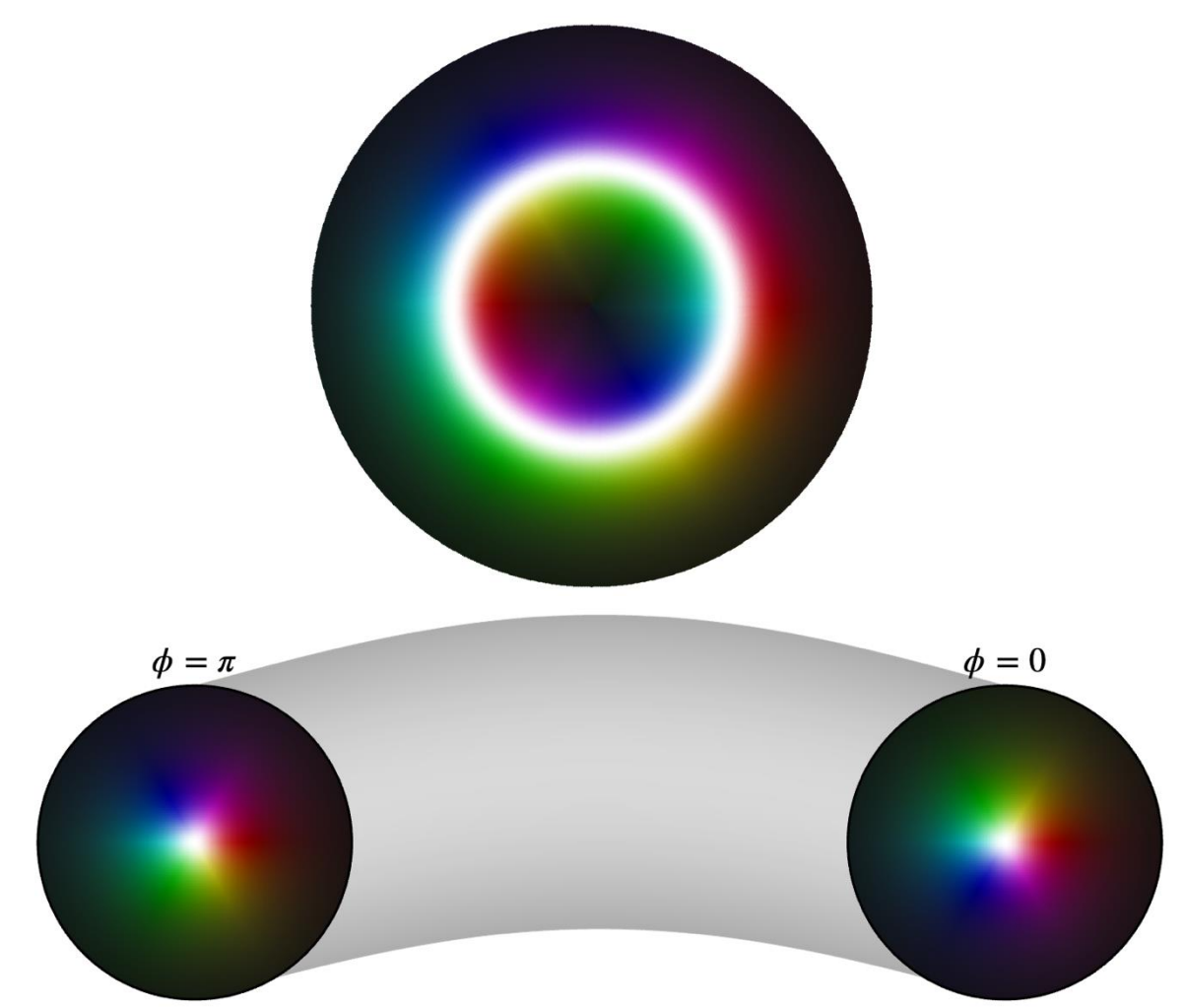


Hopfion fibers

Connecting the vectors across varying ϕ traces out Hopfion fibers



Fibration



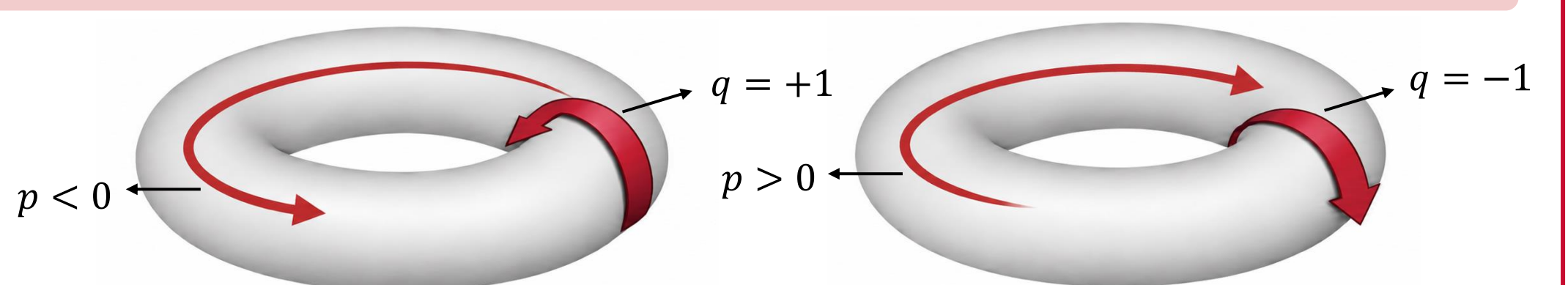
Hopfion number

Topological invariant

$$H = pq$$

p : skyrmion number of each slice

$q = \pm 1$: fiber winding direction



Conclusion

- ❑ Demonstrated skyrmion and hopfion topologies in a three-photon entangled state
- ❑ Established the tunability of the skyrmion texture
- ❑ Reconstruction of Hopfion topology in synthetic space

A foundation for exploring multiphoton topology