Thermodynamic performance of a periodically driven harmonic oscillator correlated with the baths

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I. Introduction

We consider a harmonic oscillator under periodic driving and coupled to two harmonic oscillator heat baths at different temperatures. We use the thermofield transformation with chain mapping for this setup which allows us to study the unitary evolution of the system and the baths up to a time long enough to see the emergence of periodic steady state in the system. We characterize this periodic steady state and we show that, by tuning the system and the bath parameters, one can turn this system from an *engine* to an *accelerator* or even to a *heater*. The possibility to study the unitary evolution of system and baths also allow us to evaluate the steady correlations that build between the system and the baths, and correlations that grow between the baths.

II. Methods and model

We consider a single-site bosonic harmonic oscillator coupled to two heat baths at different temperatures [Fig. 1(a)]. The harmonic oscillator has a trapping frequency $\omega_{\rm dri}(t)$.

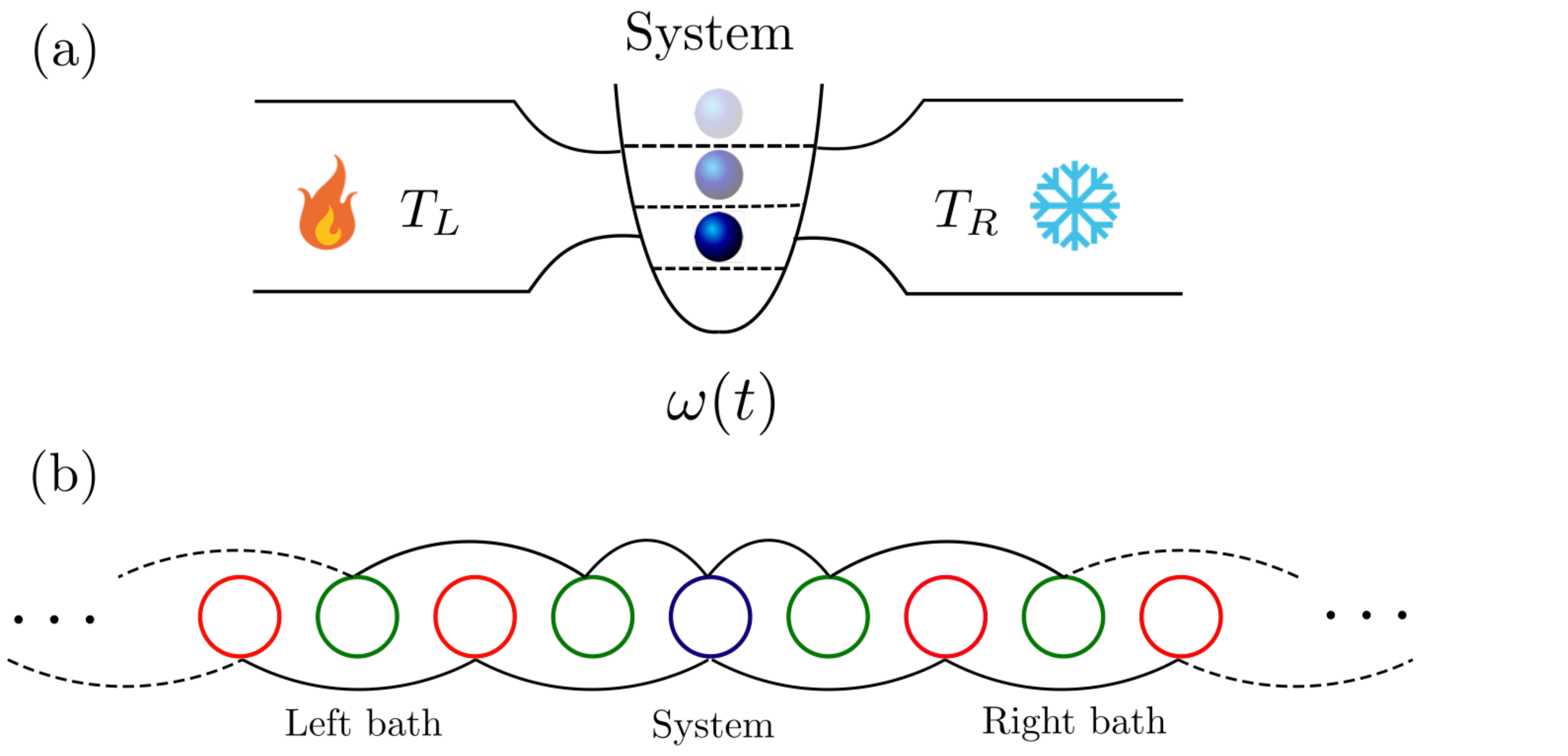


Fig. 1: (a) Schematics of a driven single bosonic harmonic oscillator coupled to two heat baths at different temperatures. (b) System coupled to the two baths after thermofield transformation [1] with chain mapping [2]: each bath is mapped to a chain with next-nearest neighbor tunneling or, equivalently, to two chains.

The totla Hamiltonian is as follows where the bath is linearly discretized [3] and then transformed as:

$$\hat{\mathcal{H}}_{\rm tot} = \hat{\mathcal{H}}_{\rm S} + \hat{\mathcal{H}}_{\rm EXT}(t) + \hat{\mathcal{H}}^L_{\rm B} + \hat{\mathcal{H}}^R_{\rm B} + \hat{\mathcal{H}}_{\rm I} \quad \text{(original Hamiltonian)}$$

$$\hat{\mathcal{H}}_{\rm S} = \hbar\omega_{\rm S} a^\dagger_{\rm S} a_{\rm S}$$

$$\hat{\mathcal{H}}_{\rm EXT}(t) = \hbar\delta\sin(\omega_{\rm dri}t) a^\dagger_{\rm S} a_{\rm S}$$

$$\hat{\mathcal{H}}^\nu_{\rm B} = \sum_k \hbar\omega_k b^{\nu\dagger}_k b^\nu_k$$

$$\hat{\mathcal{H}}_{\rm I} = \sum_{\nu=L,R}\sum_k \hbar\omega_{\rm S}\sqrt{\mathcal{J}^\nu_k}\left(a_{\rm S} b^{\nu\dagger}_k + a^\dagger_{\rm S} b^\nu_k\right).$$

Lorentzian-type bath spectral density

$$\mathcal{J}^\nu(\omega) = \frac{\gamma_\nu}{\pi}\frac{[\pi G_\nu(\omega)]^2}{(\omega-\omega^\nu_r)^2 + [\pi G_\nu(\omega)]^2} \quad (\nu = L, R)$$

$$\Downarrow$$

$$\hat{\mathcal{H}}^{\rm TC}_{\rm tot}/\hbar\omega_{\rm S} = a^\dagger_{\rm S} a_{\rm S} + (\delta/\omega_{\rm S})\sin(\omega_{\rm dri}t) a^\dagger_{\rm S} a_{\rm S} \quad \text{(after transformation)}$$

$$+\sum_{\zeta,\nu}\left[\sum_{j=1}^{N_{\rm chain}}\alpha^\nu_{\zeta,j} d^{\nu\dagger}_{\zeta,j} d^\nu_{\zeta,j} + \sum_{j=1}^{N_{\rm chain}-1}\beta^\nu_{\zeta,j}\left(d^{\nu\dagger}_{\zeta,j} d^\nu_{\zeta,j+1} + d^\nu_{\zeta,j} d^{\nu\dagger}_{\zeta,j+1}\right)\right]$$

$$+\sum_\nu\left[\beta^\nu_{1,0}\left(a^\dagger_{\rm S} d^{\nu\dagger}_{1,1} + a_{\rm S} d^\nu_{1,1}\right) + \beta^\nu_{2,0}\left(a^\dagger_{\rm S} d^{\nu\dagger}_{2,1} + a_{\rm S} d^\nu_{2,1}\right)\right].$$

III. Results

The energy current from the bath is obtained from Heisenberg EOM:

$$J^L_e = -\frac{d}{dt}\langle\hat{\mathcal{H}}^L_{\rm B}\rangle = -\frac{i}{\hbar}\left\langle\left[\hat{\mathcal{H}}_{\rm tot}, \hat{\mathcal{H}}^L_{\rm B}\right]\right\rangle$$

$$J^R_e = \frac{d}{dt}\langle\hat{\mathcal{H}}^R_{\rm B}\rangle = \frac{i}{\hbar}\left\langle\left[\hat{\mathcal{H}}_{\rm tot}, \hat{\mathcal{H}}^R_{\rm B}\right]\right\rangle.$$

$$\Rightarrow$$

$$J^L_e/J_0 = -\left(2\beta^L_{1,0}\beta^L_{1,1}{\rm Im}\langle d^{L\dagger}_{1,2} d_{\rm S}\rangle + 2\beta^L_{1,0}\alpha^L_{1,1}{\rm Im}\langle d^{L\dagger}_{1,1} a_{\rm S}\rangle + 2\beta^L_{2,0}\beta^L_{2,1}{\rm Im}\langle d^{L\dagger}_{2,2} a_{\rm S}\rangle + 2\beta^L_{2,0}\alpha^L_{2,1}{\rm Im}\langle d^{L\dagger}_{2,1} a_{\rm S}\rangle\right)$$

$$J^R_e/J_0 = \left(2\beta^R_{1,0}\beta^R_{1,1}{\rm Im}\langle d^{R\dagger}_{1,2} a_{\rm S}\rangle + 2\beta^R_{1,0}\alpha^R_{1,1}{\rm Im}\langle d^{R\dagger}_{1,1} a_{\rm S}\rangle + 2\beta^R_{2,0}\beta^R_{2,1}{\rm Im}\langle d^{R\dagger}_{2,2} a_{\rm S}\rangle + 2\beta^R_{2,0}\alpha^R_{2,1}{\rm Im}\langle d^{R\dagger}_{2,1} a_{\rm S}\rangle\right)$$

$T_L = 2T_0, 22.5T_0$
darker color for larger value

$\delta = 0.1\omega_{\rm S}, 0.5\omega_{\rm S}$
darker color for larger value

$\gamma = 0.2, 1$
darker color for larger value

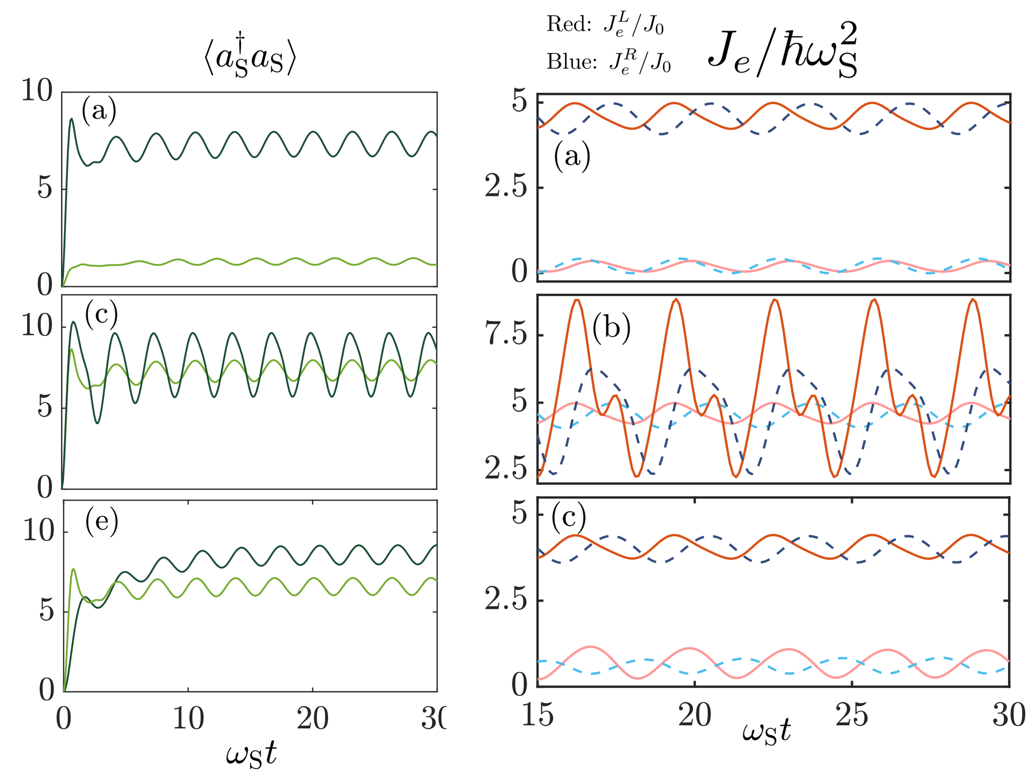
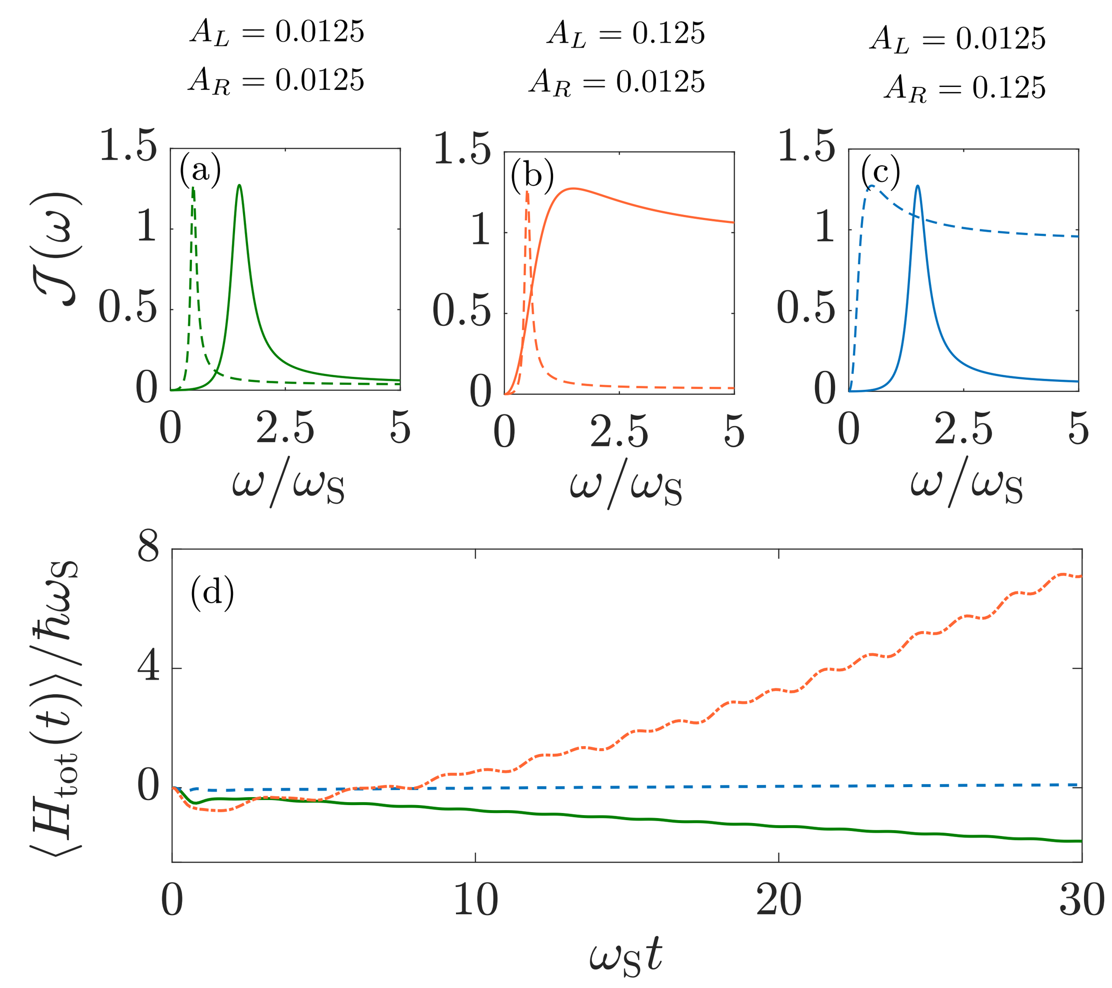


Fig. 2: (Left column) periodic system occupation and (right column) energy current: For all panels, we have used: $T_R = T_0 = \hbar\omega_{\rm S}/k_B, \omega_{\rm dri} = 0.5\omega_{\rm S}, \omega_c = 5\omega_{\rm S}, A_L = A_R = 0.0125,$ $\omega^L_t = 1.5\omega_{\rm S}, \omega^R_r = 0.5\omega_{\rm S}$.

To obtian an engine, we perform the separation of bath spectral densities:

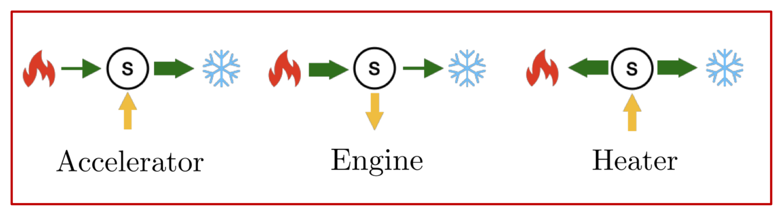


Lorentzian-type bath spectral density

The total energy of the system and two baths as a function of time.

Fig. 3: Comparison of the effect of the separation of the bath spectral densities and the total energy: from panel (a) to (c), solid curves are for the left bath spectral density, and the dashed lines are for the right bath spectral density. Other parameters used are: $T_L = 20T_0, T_R = T_0, \omega_c = 5\omega_{\rm S}, \omega_{\rm dri} = 0.5\omega_{\rm S}$, $\omega^L_r = 1.5\omega_{\rm S}, \omega^R_r = 0.5\omega_{\rm S}, \delta = 0.1\omega_{\rm S}$.

Illustration of energy transfer patterns in accelerator, engine, and heater.



To characterize the system, we compute the averaged energy current from the bath and the efficiency:

$$\bar{J}^\nu_e = (1/\mathcal{T})\int_{t_0}^{t_0+\mathcal{T}} J^\nu_e(\tau)d\tau \;\; (\nu = L, R) \qquad \eta = \frac{\bar{J}^L_e - \bar{J}^R_e}{\bar{J}^L_e} = 1 - \frac{\bar{J}^R_e}{\bar{J}^L_e}$$

$$\mathcal{T} = 2\pi/\omega_{\rm dri}$$

$\gamma = 0.2, 0.4, 1$
(darker color for larger value)

$\gamma = 0.25, 0.5, 1$
(darker color for larger value)

$\delta = 0.1\omega_{\rm S}, 0.2\omega_{\rm S}, 0.6\omega_{\rm S}$
(darker color for larger value)

$T_L = 1.5T_0, 3.75T_0, 7.5T_0$
(darker color for larger value)

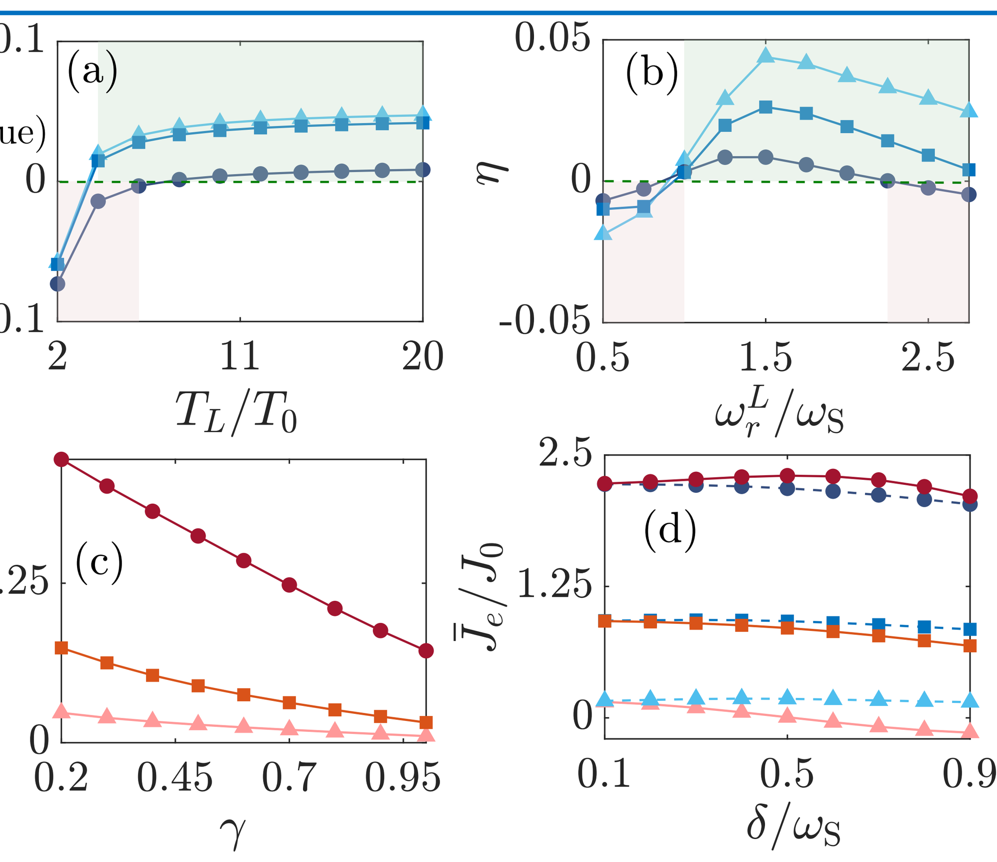


Fig. 4: Thermal machine efficiency and averaged currents: For all panels, the mutual parameters used are: $\gamma = 0.5, \omega_c = 5\omega_{\rm S}, \omega_{\rm dri} = 0.5\omega_{\rm S}, \omega^L_r = 1.5\omega_{\rm S}, \omega^R_r = 0.5\omega_{\rm S}, A_L = A_R = 0.0125$.

Finally, we can obtain the correlations [4] between baths, and between the system and the baths:

$$C(\sigma_{S+B_i}||\sigma_S\otimes\sigma_{B_i}) = {\rm Tr}\sigma_{S+B_i}\left[\log\sigma_{S+B_i} - \log(\sigma_S\otimes\sigma_{B_i})\right] \;\; (i = L, R)$$

$$C(\sigma_{B_L+B_R}||\sigma_{B_L}\otimes\sigma_{B_R}) = {\rm Tr}\sigma_{B_L+B_R}\left[\log\sigma_{B_L+B_R} - \log(\sigma_{B_L}\otimes\sigma_{B_R})\right].$$

$\sigma_T = \langle c^\dagger_i c_j\rangle$ where c_j represent either the operator a_S *or* b^ν_k in the original Hamiltonian.

$\gamma = 0.25, 0.5, 1$
(darker color for larger value)
$T_L = 20T_0, \delta = 0.1\omega_{\rm S}$

$T_L = 3T_0, 5T_0, 10T_0$
(darker color for larger value)
$\gamma = 0.25, \delta = 0.7\omega_{\rm S}$

$\delta = 0.3\omega_{\rm S}, 0.5\omega_{\rm S}, 0.7\omega_{\rm S}$
(darker color for larger value)
$\gamma = 0.25, T_L = 10T_0$

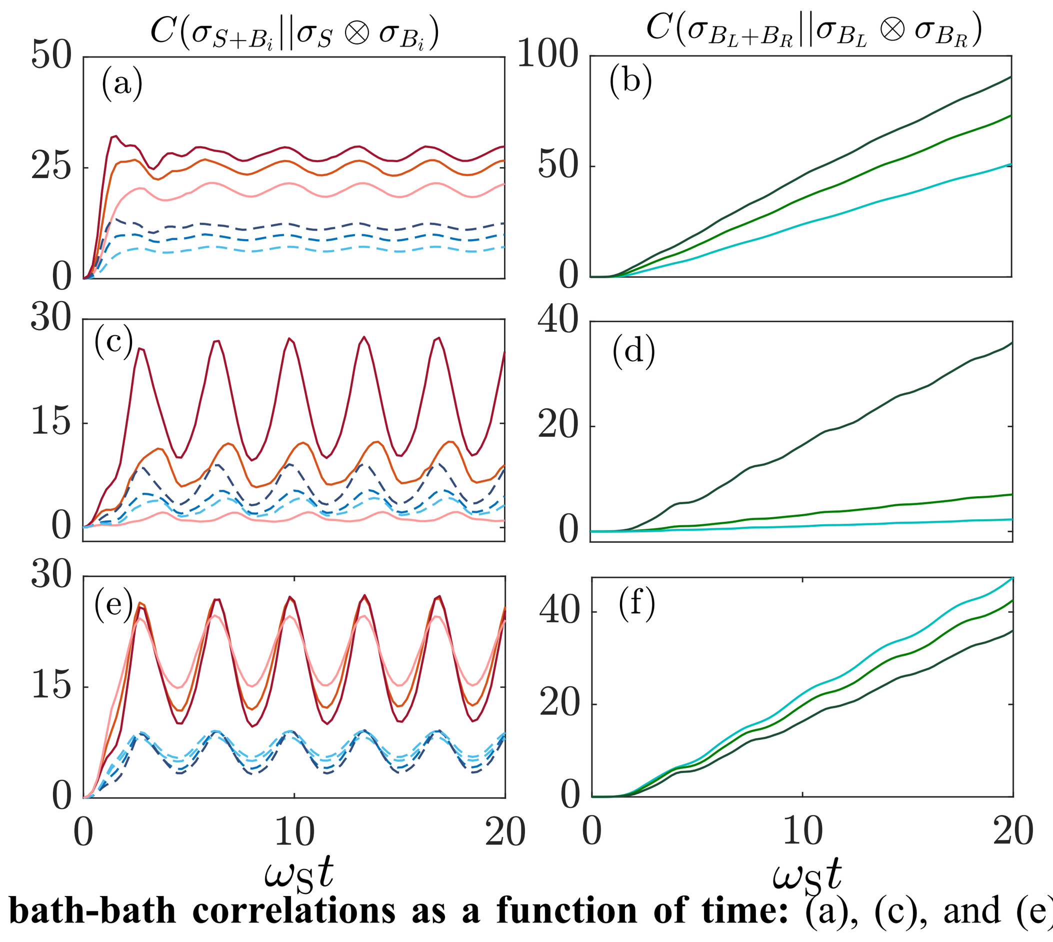


Fig. 5: Measure of system-bath and bath-bath correlations as a function of time: (a), (c), and (e) are correlations between the system and each bath, and (b), (d), and (f) are correlations between two baths. For all panels, we have used $T_R = T_0, \omega_{\rm dri} = 0.5\omega_{\rm S}, \omega_c = 2.75\omega_{\rm S}, \;\; A_L = A_R = 0.0125$, and the measurement step is 0.2.

IV. References

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