Bell nonlocality in the Universes permitting closed timelike curves

Lu Jianlong
Department of Physics, National University of Singapore
jianlong_lu@u.nus.edu

Abstract
In this paper I focus on Bell nonlocality in the Universes which permit the existence of closed timelike curves. In quantum mechanics and quantum information, Bell nonlocality is an important notion and quantum entanglement provides nontrivial ways for realizing the violation of corresponding Bell inequalities. However, in this paper I would like to argue that, in specific region of the Universes permitting closed timelike curves, the violation of Bell inequalities can be trivial and quantum entanglement is not a necessity for its realization. A necessary and sufficient condition for possibility of such trivial violation of Bell inequalities is given, which was originally proved by Carter in relevant research of general relativity. Here trivialness means the violation of Bell inequalities does not require any known communication channel or quantum entanglement.

Assumptions and Conventions

Here we have the following assumptions:
1. Einstein summation convention is assumed.
2. The "God-given" unit system $c = \hbar = G = 1$ is chosen.
3. The spacetime metric signature follows the east coast style, i.e. $(- + ++)$.
4. Greek letters vary among (between) temporal and spatial dimensions, while Latin letters take care of only spatial dimension(s).

Introduction

Bell nonlocality

The historical origin of Bell nonlocality is the influential paper by Albert Einstein, Boris Podolsky and Nathan Rosen entitled "*Can Quantum Mechanical Description of Physical Reality Be Considered Complete?*"[1] [2] Although always being mentioned together with quantum mechanics, Bell nonlocality is actually formulated independent of quantum mechanics. Roughly speaking, Bell inequality tells us what can be achieved in some specific kinds of ask-and-answer games in Einstein's classical paradise.[3] In these games (called Bell tests for obvious reason), there is a verifier and several players (more than one) and a set of rule(s) for determining scores. The players are all in the same team under the constraint that they can discuss and plan strategies (i.e., preparing shared resource) together before each round of game but cannot communicate with each others during the game. Bell nonlocality draws a limit line for possible scores, beyond which is an impossible mission for this team of players if assumption of locality holds. For better explanation, let's consider a game called Clauser-Horne-Shimony-Holt (CHSH) test. In CHSH test, there are two possible inputs (0 and 1, denoted as x if sent to Alice and y if sent to Bob) from verifier for both two players (usually named Alice and Bob as mentioned just now, due to physicists' naming tradition) and they both have two choices for answer (± 1, denoted as a_x if answered by Alice for the input x, and b_y if answered by Bob for the input y). This game will be repeated for many rounds and the final score is

$$S = \langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle. \tag{1}$$

By listing out all the $2^4 = 16$ possible pre-determined answer strategies, it is easy to find that

$$|S| \leq 2. \tag{2}$$

In other words, if Alice and Bob together choose a specific quadruple $(a_0, a_1; b_0, b_1)$ before each round starts, their final score can only lie between ± 2. furthermore, fortunately, there is a Fine's Theorem stating that all local statistics (i.e., statistics that can be achieved by blocking communication and signaling between players, mathematically defined by

$$P(a, b|x, y) = \int d\lambda Q(\lambda) P(a|x, \lambda) P(b|y, \lambda) \tag{3}$$

with $Q(\lambda)$ representing some probability distribution from players' sampling on their shared resource λ) can be realized by pre-determined output strategy, which ensures that the above 16 quadruples are all we need to worry about in CHSH test.

Closed timelike curves

In Einstein's general relativity, all objects with nonvanishing mass travel along timelike curves in spacetime. However, there exist peculiar situations in which a timelike curve form a "loop" by itself, called closed timelike curve. The notion of closed timelike curve was first quantitatively realized by Gödel in 1949.
Although in the standard model of modern cosmology (Lambda-CDM model) based on general relativity we can foliate the spacetime into spacelike hypersurfaces and parametrize them with unambiguous real numbers called cosmic time, there still exist solutions to Einstein field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{4}$$

which permit the existence of closed timelike curves, such as Gödel metric, Kerr spacetime and Tipler cylinder.

Examples of Universes permitting closed timelike curves

Gödel spacetime

Gödel metric can be written as the following form

$$ds^2 = a^2(-dx^0dx^0 + dx^1dx^1 - \frac{e^{2x_1}}{2} dx^2dx^2 + dx^3dx^3 - 2e^{x_1}dx^0dx^2) \tag{5}$$

with nonzero real constant a which is related to the matter density and cosmological constant appearing in the Gödel universe.[4] It is straightforward to show that the contents in Gödel spacetime is matter with density $\rho_M = \frac{1}{8\pi G a^2}$ and cosmological constant $\Lambda = -\frac{1}{2a^2}$ by substituting it back into Einstein field equation.
In Gödel spacetime, it can be shown that *for any two distinct events there exists a closed timelike curve passing through them.*

Kerr spacetime

Kerr metric [5], found in 1963, is one of the four famous exact black hole solutions of Einstein field equation, which can be written in Boyer–Lindquist coordinates as the following form

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr}{r^2 + a^2\cos^2\theta}\right)dt^2 - \frac{4Mra\sin^2\theta}{r^2 + a^2\cos^2\theta} dt d\phi + \left(\frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2}\right)dr^2 + (r^2 + a^2\cos^2\theta)d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{r^2 + a^2\cos^2\theta}\right)\sin^2\theta d\phi^2 \end{aligned} \tag{6}$$

where M is the mass and $a = \frac{J}{M}$ with angular momentum J. (Note that here we are using the convention $c = \hbar = G = 1$)
It is also easy to show that there exist closed timelike curves in the region $r < 0$ within the inner event horizon of the maximally extended Kerr spacetime.

Tipler cylinder

Tipler cylinder is a hypothetical equipment which can help to create closed timelike curves.[6] The curvature of the spacetime is generated by the infinitely long Tipler cylinder which is rapidly rotating around its axis. The relevant solution to Einstein field equation was first found in 1936 by van Stockum, which can be written in the $3 + 1$-dimensional cylindrical coordinates as follows

$$ds^2 = -Fdt^2 + Ld\phi^2 + 2Mdtd\phi + H(dr^2 + dz^2) \tag{7}$$

in which the coordinate z is along the rotating axis, the coordinate r is radially outwards with respect to the rotating axis, and coordinate ϕ is the angle. Note that the coefficients F, L, M, H are not constants but functions of r, first three of which are constrained by the condition $FL + M^2 = r^2$.
Given a as the angular velocity of the Tipler cylinder, we can prove that any two events are connected by timelike curves when the boundary of the cylinder R satisfying $aR > \frac{1}{2}$.

Bell test in Universes satisfying Carter's condition

Carter's causality theorem tells us that, *for a connected, time-oriented spacetime with a timewise orthogonally transitive Abelian isometry group if there does not exist a covariant vector in the Lie algebra such that the corresponding differential form in the surface of transitivity is everywhere well-behaved and everywhere timelike then there must exist both future and past directed timelike curves between any two events in this spacetime.*[7][6]
Imagine a physicist doing a Bell test in the lab. He chosen two spacetime points (i.e., events) in the lab and by small scale measurement done within the lab he believed that these two events are spacelike separated. Assuming all known loopholes are closed, then this physicist will need quantum entanglement to achieve violation of corresponding Bell inequality. In other words, in the assumption of loophole-free and trivial spacetime, quantum entanglement seems the necessary condition for violation of Bell inequality. However, if this physicist actually lives in a spacetime satisfying Carter's condition, then those events which seem to be spacelike separated from each other are in fact causally connected by timelike curves. Along such timelike curve, event A can send information to event B, which is spacelike separated from A if we focus only on a small scale part of the whole spacetime. Since participants at A and B can share the information about inputs from the physicist, they can of course conspire to fool the physicist by violating his expected inequality. Thus the independence condition for Bell inequality does not necessarily hold. This brings the result that the Bell inequality *may* be violated trivially without any help from quantum entanglement.

Why to think about it?

1. Physicists are always "local". In other words, a ideal point-like physicist lives in and plays with a trivial Minkowskian neighborhood, whose conclusion from experiment may be not suitable if larger scale structure of spacetime is considered.
2. As a local theory, general relativity itself can indeed exhibit "nonlocality".
3. A purely classical system can exhibit some kind of "entanglement".

Conclusion

Unlike the situation in quantum mechanics and quantum information which usually assume the background spacetime to be the Minkowskian one, we have seen that in more general spacetimes which permit the existence of closed timelike curves Bell inequalities can be violated trivially without necessarily referring to quantum entanglement, at least within some specific regions.
However, this should not be merely understood as a possible loophole of Bell test. There is something deeper. In the above analysis we can see that, at least in some specific regions of the Universe permitting the existence of closed timelike curves, the absolute distinction between locality and nonlocality is gone. Thus a field theory which encodes the idea of locality may have an effective equivalent expression based on action-at-a-distance.
According to Lambda-CDM model closed timelike curves do not play a role in the large scale structure of our Universe. However, if we observe trivial violation of Bell inequalities, i.e., without any known communication channel or quantum entanglement, then the possibility of closed timelike curves existing in small scale structure should not be abandoned without further justification.

Acknowledgements

I would like to thank Prof. Oh sincerely for his patience in my (possibly trivial) idea and help in pushing myself to make the statement clearer.

References

[1] A. Einstein, B. Podolsky, and N. Rosen. *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*. Physical Review, 47: 777–780. (1935)

[2] J. S. Bell. *On the Einstein Podolsky Rosen Paradox*. Physics. 1 (3): 195–200. (1964)

[3] V. Scarani. *Bell Nonlocality*. Oxford University Press.

[4] K. Gödel. *An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation*. Rev. Mod. Phys. 21, 447. (1949)

[5] R. P. Kerr. *Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics*. Phys. Rev. Lett. 11, 237. (1963)

[6] F. J. Tipler. *Rotating Cylinders and the Possibility of Global Causality Violation*. Physical Review D. 9 (8): 2203–2206. (1974)

[7] B. Carter. *Global Structure of the Kerr Family of Gravitational Fields*. Phys. Rev. 174, 1559. (1968)